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Abstracts

On the effect of higher order derivatives of initial data on the blow-up set for a semilinear heat equation

Yohei Fujishima (Osaka University)

Abstract

We consider a semilinear heat equation with large initial data, and characterize the location of the blow-up set. For the case the initial data is of the form $\lambda\varphi(x)$ with large parameter λ, it has been proved that the solution blows up near the maximum points of φ. Furthermore, if φ has several maximum points, then the blow-up set is affected by $\Delta\varphi$. However, it seems difficult to obtain further information on the blow-up set if $\Delta\varphi$ has same value at the maximum points of φ. In this talk, we consider the case the initial data is of the form $\lambda + \varphi(x)$, and study the location of the blow-up set. In particular, we study the relationship between the blow-up set and higher order derivatives of the initial data.

New patterns of travelling waves in the generalized Fisher-Kolmogorov equation

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Abstract

We will briefly discuss the mathematical model for the co-existence of three possible genotypes A_1A_1, A_1A_2, A_2A_2, called *"A Selection-Migration Model in Population Genetics"* by W. H. FLEMING in 1975 (J.Math.Biology). This is a classical model with the "regular" diffusion modelled by the (linear) Laplace operator. In this model, a travelling wave can model solely the mixed genotype A_1A_2; neither of the pure genotypes A_1A_1, A_2A_2 can exist alone in any open subset of the spatial domain $\mathbb{R} = (-\infty, \infty)$. However, if a kind of "degenerate" diffusion is introduced into the model, this time modelled by the (nonlinear) p-Laplace operator Δ_p, $\Delta_p u \stackrel{\text{def}}{=} \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, the coexistence of *all three* genotypes, A_1A_1, A_1A_2, A_2A_2, becomes possible if $p \neq 2$. We will show this by a simple asymptotic analysis of the travelling wave $u(x,t) = U(x - ct)$, where $U : \mathbb{R} \to \mathbb{R}$ is strictly monotone decreasing and continuously differentiable with $U' < 0$ on an interval in $\mathbb{R}$. By finding a *first integral* for the second-order differential equation for U, we obtain a nonlinear *overdetermined* problem for U with a free parameter c, the speed of propagation. This problem is uniquely solvable for U and the parameter c; $c = c^* \in \mathbb{R}$ is this unique critical value for c. Of course, the reaction function

[1]Joint work with PAVEL DRÁBEK, Department of Mathematics and N.T.I.S. (Center of New Technologies for Information Society) University of West Bohemia, Plzeň, Czech Republic

$f(s)$ must satisfy certain asymptotic conditions near its two "stable" zeros ± 1. We will infer from the asymptotic analysis of the travelling wave U near the points $s = \pm 1$, that the three genotypes may co-exist even for the "regular" diffusion ($p = 2$), provided the reaction function $f(s)$ has a "suitable" asymptotic behavior near $s = \pm 1$.

In the second lecture we construct the nonlinear semi-flow generated by our model, i.e., by the quasilinear Fisher-Kolmogorov equation. We make use of a natural approximation method using an increasing sequence of bounded intervals, a local Hölder regularity result, and the locally uniform convergence of the sequence of approximating solutions to the desired solution on the whole of $\mathbb{R}$. Finally, using a Ljapunov functional, we prove the convergence of any weak solution (with some natural restrictions on the initial data) to a travelling wave. The kind of Ljapunov functional we use has been constructed for $p = 2$ in the work by P. C. Fife and J. B. McLeod in 1977.

Keywords: Fisher-Kolmogorov equation, travelling waves, nonlinear diffusion, nonsmooth reaction function, comparison principle, convergence to a travelling front

2010 Mathematics Subject Classification: Primary 35Q92, 35K92; Secondary 35K55, 35K65

Interfaces in the Fisher equation and a Hamilton-Jacobi equation

Eiji Yanagida (Tokyo Institute of Technology)

Abstract

We consider the dynamics of interfaces in the Fisher-KPP equation

$$\varepsilon u_t = \varepsilon^2 \Delta u + u(1-u) \quad \text{on } \mathbb{R}^N,$$

where $\varepsilon > 0$ is a small parameter. Solutions of this equation exhibit interfaces that correspond to transition layers from the trivial steady state to a positive steady state. If an initial value decays rapidly in space, then the interface moves with a constant speed that is equal to the minimal speed of traveling fronts in one-dimensional space. On the other hand, it is known that if an initial value decays slowly, the interface may move in a rather irregular way. In this talk, we show that in the limit as $\varepsilon \to 0$, the dynamics of interfaces for slowly decaying initial data can be described as a level set of a Hamilton-Jacobi equation

$$v_t = |\nabla v|^2 + 1.$$

We also discuss properties of solutions of the Hamilton-Jacobi equation. This is a joint work with Hirokazu Ninomiya (Meiji University).